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A structural reliability-based approach to prestressed concrete design

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Abstract

The design of a prestress configuration for flexure is governed by the serviceability limit state of cracking. Structural design standards provide guidelines for design of prestressed concrete by specification of allowable stresses and a set of partial factors applicable for the different limit states considered. The calibration of partial factors for a target reliability level is generally only performed for the ultimate limit state, whereas all partial factors for the verification of the serviceability limit state are set equal to unity in the Eurocode suite of design standards. This study demonstrates that these partial factors are not reliability-based which results in sub-optimal reliability performance. The reliability performance of these factors is contrasted against the reliability performance of a set of partial factors for SLS design of prestressed concrete elements calibrated using the design value method described by the Eurocodes in EN 1990:2002. The developed set of partial factors presents an improvement of the current state of the art with the potential for optimised durability of designs.

Keywords: Prestressed concrete, Serviceability limit state, Structural reliability, Durability

1. INTRODUCTION

Modern design standards are based on the limit states design philosophy which is frequently calibrated using the theory of structural reliability. These standards provide a procedure for design of prestressed concrete elements by specification of allowable stresses and a set of partial factors applicable for the different limit states considered. The allowable stress limits are to be satisfied at transfer of prestress force and during service. For an appropriately scaled cross section, these allowable stresses can be written as four stress inequalities which delimit a domain of feasible prestress configurations expressed graphically as a Magnel diagram [1].

Partial factors are calibrated based on the associated statistical characteristics of the different loading and resistance parameters for a specified target reliability level. The calibration of partial factors for a target reliability level is generally only performed for the ultimate limit state, whereas all partial factors for verification of the serviceability limit state are set equal to unity in the Eurocode suite of design standards [2]. Variable uncertainty is, therefore, addressed only by the specified characteristic values and may result in a greater tolerable probability of failure in design than is specified by the target reliability level. This specification is especially a concern when structural design is governed by the serviceability limit state, as for prestressed concrete design, as it may lead to unsatisfactory structural performance.

In this paper the reliability performance of the prestressed concrete serviceability limit state design procedure of the Eurocodes[3] and a set of partial factors calibrated using the design value method (DVM) are contrasted. A single span concrete girder bridge with composite prestress concrete sections is assumed as the reference structure. An analysis model is developed to evaluate the reliability along the boundaries of the Magnel diagram.

2. FUNDAMENTAL STRUCTURAL RELIABILITY BACKGROUND

Structural reliability concerns the probabilistic measure of structural safety. To ensure safety in design, a margin of safety is provided which is quantified by means of a performance function. The performance function for the SLS is defined as $g(\boldsymbol{x},\boldsymbol{a}) = L - E$ where $\boldsymbol{x}$ and $\boldsymbol{a}$ are vectors of random variables and deterministic parameters respectively [4] and L and E are the limiting design value and load effects respectively. The reliability of a structure is expressed probabilistically in terms of a probability of failure which denotes the probability of $g(\boldsymbol{x},\boldsymbol{a}) < 0$. When multiple limit states are significant in determining the probability of failure (as for prestressed concrete) the probability of failure is determined as the probability of violation of the union of the limit states ($\bigcup_i g_i(\boldsymbol{x},\boldsymbol{a}) < 0$). The probability of failure is then determined as,

$$p_f = p\left(\bigcup_i g_i(\boldsymbol{x},\boldsymbol{a}) < 0\right) = \int_{\bigcup_i g_i(\boldsymbol{x},\boldsymbol{a})<0} f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} \tag{1}$$

where $f_{\boldsymbol{X}}(\boldsymbol{x})$ is the multivariate probability density of $\boldsymbol{X}$ [4]. The probability of failure is frequently rewritten as an associated reliability index $\beta = -\Phi^{-1}(p_f)$. The exact closed-form solution of Eq 1 is rarely achievable. In this study Eq 1 is integrated by employing the Monte Carlo method. This method generates sampling sets of each of the random variables based on their associated distributions. Using these sampling sets, many realisations of the performance function are determined from which the number of failures are counted to determine the probability of failure.

3. PROBABILISTIC ANALYSIS MODEL

To evaluate the reliability performance of the SLS design procedure a probabilistic analysis model is developed.

3.1 Reference Structure

The reference structure selected for the analysis model is a 3-meter-wide single span, simply supported concrete girder bridge. The bridge has a span length of 20 meters and is comprised of three equally spaced precast beams with a cast in situ deck forming composite prestressed concrete sections. The precast beam profiles are based on the geometry of standardised AASHTO I-beam profiles[5]. The geometry to be considered at transfer and service stresses for the generation of the Magnel diagram is provided in Figure 1.

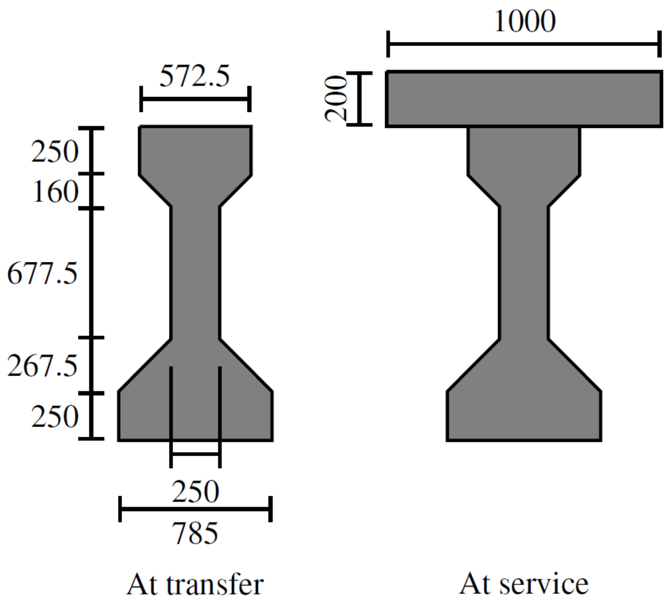


Figure 1: Cross section used in the study (units in mm)

The reference structure is to be designed for traffic loading in accordance with Load Model 1 (LM1) specified in EN 1991-2:2003[6]. Accounting for transverse load distribution, the worst case characteristic applied midspan bending moment per beam for the 20m span length is 2959 kN.m. A C50/60 concrete class described in EN 1992-1-1:2004 [3] is adopted for which the characteristic compressive strength at a 7-day concrete age at transfer is 40.94 MPa. The stress limits at transfer and service as described in EN 1992-1-1:2004 [3] are used. To determine the load effects owing to self-weight of the composite section, the nominal density of concrete is assumed equal to 24 kN/m^3. Further, it is assumed that the precast sections are post-tensioned and the total time-dependent prestress losses are 25% of prestress force at transfer.

3.2 Reliability Verification

The reliability performance of prestressed concrete can be evaluated by the definition of four performance functions based on the four stress inequalities to be satisfied. These performance functions are,

$$g_1(\boldsymbol{x},\boldsymbol{a}) = \sigma_{t,t} - \left[\frac{P_t}{A_b} + \frac{P_t e}{Z_{top,b}} + \frac{\theta_{E,M} M_b}{Z_{top,b}}\right] \tag{2}$$

$$g_2(\boldsymbol{x},\boldsymbol{a}) = \left[\frac{P_t}{A_b} + \frac{P_t e}{Z_{bot,b}} + \frac{\theta_{E,M} M_b}{Z_{bot,b}}\right] - \sigma_{c,t} \tag{3}$$

$$g_3(\boldsymbol{x},\boldsymbol{a}) = \left[\frac{\eta P_t}{A_b} + \frac{\eta P_t e}{Z_{top,b}} + \frac{\theta_{E,M}M_b + \theta_{E,M}M_F}{Z_{top,b}} + \frac{\theta_{E,M}M_L}{Z_{top,cb}}\right] - \sigma_{c,s} \quad (4)$$

$$g_4(\boldsymbol{x},\boldsymbol{a}) = \sigma_{t,s} - \left[\frac{\eta P_t}{A_b} + \frac{\eta P_t e}{Z_{bot,b}} + \frac{\theta_{E,M}M_b + \theta_{E,M}M_F}{Z_{bot,b}} + \frac{\theta_{E,M}M_L}{Z_{bot,cb}}\right] \quad (5)$$

In the above performance functions M_b, M_F and M_L are the moments generated by precast beam self-weight, cast in situ deck self-weight and traffic loading respectively. To integrate Eq 1 for a given prestress configuration, Monte Carlo simulations generating 5×10^7 trials of each of the four performance functions is performed. A trial is deemed to have failed if any of the performance functions are violated. The random variables defining the above performance functions are characterised by the probabilistic models and population parameters in Table 1.

Table 1: Probabilistic models of basic variables

Variable, X	Unit	Distribution	Mean, μ_x	CoV, δ_x	Reference
Self-weight	kN/m^3	Normal	24	0.04	[7]
Dimensions	mm	Normal	Varies	0.03	[8]
Initial compressive strength, $f_{cm,i}$	MPa	Log-normal	58.08	0.18	[9]
Ultimate compressive strength, $f_{cm,u}$	MPa	Log-normal	70.94	0.18	[9]
Concrete tensile strength, f_{ct}	-	Log-normal	$1.05f_{ctm}$	0.15	[10]
Prestress loss, ΔP_t	-	Normal	$0.25P_t$	0.3	[11]
Prestress force, P_t	kN	Normal	Varied	0.015	[12]
Maxima traffic load effect (50-y), M_L	kN.m	Gumbel	2655	0.061	[8]
Eccentricity, e	mm	Normal	Varied	0.015	[12]
Moment Model Uncertainty, $\theta_{E,M}$	-	Log-normal	1.0	0.1	[11]
Length, L	m	-	-	-	-

4. PARTIAL FACTOR CALIBRATION

The *fib* Model Code 2010 [13] and EN 1990:2002 [2] detail an approach for developing partial factors known as the design value method (DVM). This method is frequently used for determining partial factors for the ultimate limit state. To enable the use of the design value method to determine partial factors for SLS, a significant assumption is made that the sensitivity factors as developed by König and Hosser [14] are applicable for the serviceability limit state. These sensitivity factors assume the resistance (or limiting design value for the case of SLS) and load effect terms to have comparable coefficients of variation which may not necessarily be the case for SLS prestressed concrete design.

The target reliability for irreversible serviceability is specified in EN 1990:2002 [2] as $\beta_{t,SLS} = 1.5$ for a 50-year reference period. Making use of the design value method, the partial factors for SLS design of prestressed concrete structures are evaluated for a target reliability index of 1.5. These calculations are not detailed in this paper but are based on the equations provided in the *fib* Model Code 2010 [13]. This provides a set of partials factors aimed at satisfying the target reliability as a minimum. This set of partial factors is provided in Table 2.

Table 2: Partial factors for irreversible SLS target reliability (rounded off conservatively)

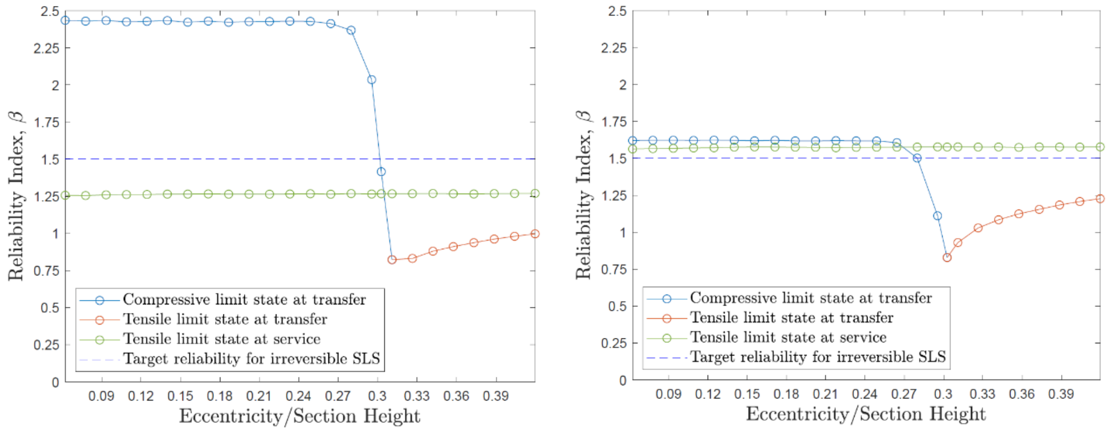
Variable	Partial Factor	Value
Concrete compressive strength	γ_c	0.92≈0.95
Concrete tensile strength	γ_{ct}	1.14≈1.15
Prestress force	γ_p	Favourable: 0.99≈1.0 Unfavourable: 1.01≈1.0
Prestress loss	γ_{pl}	Favourable: 0.85 Unfavourable: 1.13≈1.15
Permanent actions	γ_g	Favourable: 0.98 Unfavourable: 1.03
Model uncertainty	γ_{Ed}	Favourable: 0.95 Unfavourable: 1.05
$\gamma_G = \gamma_{Ed} \times \gamma_g$	γ_G	Favourable: 0.93≈0.9 Unfavourable: 1.08≈1.1
Variable actions	γ_q	0.95
$\gamma_Q = \gamma_{Ed} \times \gamma_q$	γ_Q	0.99≈1.0

5. RESULTS

Based on the loading and resistance parameters of the reference structure, the Magnel diagram delimiting the feasibility domain is generated. Dividing the feasibility domain into a grid of points and evaluating the reliability index at each point provides an indication of the performance of the design method. The lowest reliability indices are achieved at the boundaries of the Magnel diagram's feasibility domain with increasing reliability as distance from the boundaries increases. For the composite beam configuration used, only three of the four limit states (and their associated Magnel diagram equations) have significance in defining the feasibility domain for the relevant range of eccentricities. These boundaries are the compressive and tensile stress limit states at transfer, which together forms the Magnel diagram boundary with the lowest absolute value of $1/p_t$, and tensile stress limit state at service, which forms the Magnel diagram boundary with greatest absolute value of $1/p_t$. The figures below detail the reliability achieved along the boundaries of the Magnel diagram.

Figure 2(a) demonstrates the reliability performance of the design procedure specified in the Eurocodes. Use of partial factors equal to 1.0 for all the variables defining the limiting design value and action effect with the resistance factors, r_{sup} and r_{inf}, results in inconsistent and unsatisfactory reliability performance. The boundaries formed by the tensile stress limit states at transfer and service yields reliability performance below the target, $\beta_{t,sls} = 1.5$. Figure 2(a) also shows a diminishing reliability performance of the tensile stress limit state at transfer boundary as the eccentricity decreases. In contrast the boundary formed by the compressive stress limit state at transfer provides a reliability performance which significantly exceeds the target.

Figure 2(b) demonstrates the reliability performance when using the partial factors calibrated using the design value method. The use of these factors significantly improves the control of achieved reliability along the boundaries of the Magnel diagram when compared to the results when using the Eurocode prescribed factors. The boundaries formed by the compressive stress limit state at transfer and tensile stress limit state at service yield similar reliability performance, $\beta \approx 1.6$, which satisfies the target. However, the reliability achieved by the boundary formed by the tensile stress limit state at transfer fails to achieve the target reliability and also demonstrates a reduction in calculated reliability as the eccentricity decreases.



(a) Eurocode Factors

(b) DVM partial factors

Figure 2: Reliability performance of Magnel diagram boundaries

6. DISCUSSION

The significance of the tensile stress limit state at transfer attaining a reliability level lower than the target depends on the interpretation of the reference period of 50-years associated with the target reliability. It could be argued that transfer is an instantaneous event and as a result it should be associated with a lower target reliability. However, this study makes the case that the random variables defining the limit states at transfer are not time-dependant and therefore specifying the same target reliability for transfer and service is appropriate.

When using the developed set of partial factors, the failure to meet the reliability level of the tensile stress limit state at transfer indicates that this limit state is sensitive to variations of some of the random variables which are not captured by the partial factors. Further, it is clear that as eccentricity decreases the reliability achieved at the boundary reduces which shows that the sensitivity to these random variables increases with a decrease in eccentricity. This phenomenon is explored by determining the direction cosines, or sensitivity factors, of the design point which describe the sensitivity of the reliability index to the variations of the different random variables. It is found that the tensile stress limit state at transfer is highly sensitivity to geometric variations. This can be acknowledged by the introduction of a factor accounting for geometric uncertainties γ_{Rd2} in determining the partial factor for concrete tensile strength. Caspeele *et al*. [8] provides a value of $\gamma_{Rd2} = 1.10$ for concrete geometry in line with the JCSS Probabilistic Model Code [11]. The partial factor for concrete tensile strength is then determined as $\gamma_{Ct} = \gamma_{Rd} \times \gamma_{ct}$ where the model uncertainty factor for material properties is $\gamma_{Rd} = \gamma_{Rd1} \times \gamma_{Rd2} = 1.10$. For a target reliability index for the irreversible serviceability limit state of $\beta_t = 1.5$, $\gamma_{Ct} = 1.10 \times 1.14 \approx 1.25$. Using the partial factors provided in Table 2 and γ_{Ct}, the reliability performance of the Magnel diagram boundaries is determined and plotted for the range of eccentricities as shown in Figure 3.

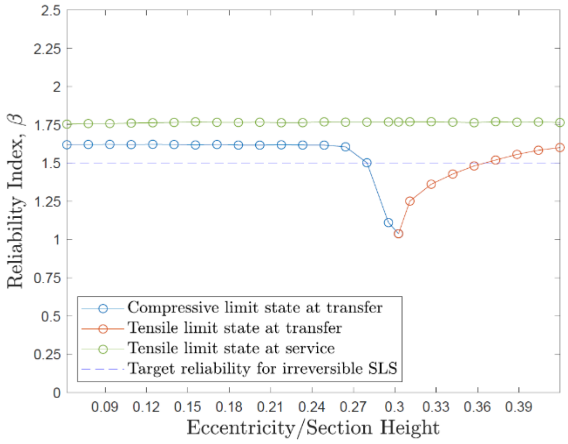


Figure 3: Reliability performance of Magnel Diagram boundaries for DVM partial factors with updated partial factor for concrete tensile strength

Figure 3 illustrates that use of the partial factor γ_{Ct} leads to a significant improvement of the reliability performance of the boundary of the Magnel diagram enforced by the tensile stress limit state at transfer. At maximum practical eccentricity, the boundary of the Magnel diagram developed by the tensile stress limit state at transfer has satisfactory reliability performance. However, the reliability performance still decreases with a reduction in eccentricity due to the change in the balance of the limit state sensitivity to random variables, resulting in geometric deviations becoming more significant.

A characteristic of the reliability performance of the Magnel diagram demonstrated by Figures 2 and 3 is that there is a steep drop in the achieved reliability along the boundaries defined by the tensile and compressive stress limit state at transfer at eccentricities approaching the eccentricity at which the two boundaries intersect. This steep drop in achieved reliability can be explained by considering that the analysis model inspects a grid of points within the feasibility domain. At each grid point, a Monte Carlo analysis generates many trials with each trial being associated with a sampling set of the vector of random variables. The four limit states are evaluated for each trial and if one of the limit states fail, the trial is deemed to have failed. In this way the probability of failure demonstrates the union of the four failure modes. At prestress design configurations located on the feasibility domain close to where the two boundaries intersect (which includes those points on the boundaries nearing this point), two different failure mechanisms become significant and the likelihood that the Monte Carlo simulations fail by either exceeding the tensile or compressive stress limits increase. As a result, the total number of failures increases on the boundaries approaching these points leading to a greater failure probability and lower reliability index.

The reliability performance deficit of design configurations located on the Magnel diagram near where the Magnel diagram boundaries intersect is concerning. However, this deficit cannot be treated directly with the partial factor framework without significantly increasing the reliability achieved along the boundaries. Increasing the reliability achieved along the boundaries above the target is unfavourable as these target reliability indices have been derived to minimise the economic cost in general for a specific structural class. It is prudent to rather recommend that these locations along the boundaries of the Magnel diagram should be avoided and a design at the maximum practical eccentricity is to be selected.

7. CONCLUSIONS

The partial factors developed in this study using the design value method (DVM) provide a significant improvement on the current state of the art provided in the Eurocodes (EC). Comparing Figure 2(a) to Figure 3 shows enhanced reliability performance at the compressive stress limit state at transfer ($\beta_{EC} = 2.45$ vs $\beta_{DVM} = 1.6$), tensile stress limit state at transfer ($\beta_{EC} = 1.0$ vs $\beta_{DVM} = 1.6$) and tensile stress limit state at service ($\beta_{EC} = 1.25$ vs $\beta_{DVM} = 1.75$) with values closer to the target $\beta_{t,SLS} = 1.5$.

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